# RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

**M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2024**
**SECOND YEAR (BATCH 2022-24)**
**MATHEMATICS**
**Paper : CC 18**

Date : 17/05/2024
Time : 11.00 am – 1.00 pm
Full Marks : 50

Answer **all** questions, maximum you can score is 50. Symbols having usual meaning.

1. Reduce the following equation
$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+5\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}-3u=0$ to its canonical form. [7]

2. Deduce Lagrange's identity for the linear PDE
$Lu\equiv\sum_{i,j=1}^{n}a_{ij}\frac{\partial^2 u}{\partial x_i\partial x_j}+\sum_{i=1}^{n}b_i\frac{\partial u}{\partial x_i}+cu=0$ , where $a_{ij}=a_{ji}$ and $a_{ij},b_i,c$ are functions of $x_1,x_2,.................x_n$ having continuous second order partial derivatives in $D\subset E^n$. Also find the condition for L to be self adjoint. [7]

3. Find the solution in closed form of the Interior Neumann Problem :
PDE : $u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0, 0\le r\le a,\ 0\le\theta\le 2\pi$
B.C. : $\frac{\partial u(a,\theta)}{\partial r}=g(\theta), 0\le\theta\le 2\pi.$ [7]

4. Find the steady state temperature distribution in a rectangular plate of sides $a$ and $b$ insulated at the lateral surface and satisfying the boundary conditions $u(0,y)=u(a,y)=0$ for $0\le y\le b$ and $u(x,b)=0,\ u(x,0)=x(a-x)$ for $0\le x\le a$ . [6]

5. Deduce Helmholtz's First Theorem from three dimensional wave equation. [7]

6. A homogeneous thermally conducting cylinder occupies the region $0\le r\le a,\ 0\le\theta\le 2\pi, 0\le z\le h$ where $r,\theta,z$ are cylindrical co-ordinates. The top $z=h$ and the lateral surface $r=a$ are held at $0^{\circ}$, while the base $z=0$ is held at $100^{\circ}$. Assuming that there are no sources of heat generation within the cylinder, find the steady – temperature distribution within the cylinder. [8]

7. Find the general solution of $\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\varphi^2}$ by the method of separation of variables, where $\alpha$ is a constant. [8]

8. Solve : $\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), 0\le x\le a,\ 0\le y\le b, t\ge 0$
B.C. : $T(0,y,t)=T(a,y,t)=0, 0\le y\le b,\ t\ge 0$
$T(x,0,t)=T(x,b,t)=0, 0\le x\le a,\ t\ge 0$
with initial condition $T(x,y,0)=f(x,y), 0\le x\le a,\ 0\le y\le b$ . [5]

9. Solve the boundary value problem $\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial t}=-8t, t>0, x>0$ with the conditions $u=0$ when $t=0$, $x>0$ and $u=2t^2$ when $x=0,\ t>0$. [5]

———— × ————